

Predicting CP violation in Deviation from Tri-bimaximal mixing of Neutrinos

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Abstract

We study the CP violation in the deviation from the tri-bimaximal mixing (TBM) of neutrinos. We examine non-trivial relations among the mixing angles and the CP violating Dirac phase in the typical four cases of the deviation from the TBM. The first two cases are derived by the additional rotation of the 2-3 or 1-3 generations of neutrinos in the TBM basis. The other two cases are given by the additional rotation of the 1-3 or 1-2 generations of charged leptons with the TBM neutrinos. These four cases predict different relations among three mixing angles and the CP violating Dirac phase. The rotation of the 2-3 generations of neutrinos in the TBM basis predicts $\sin^2 \theta_{12} < 1/3$, and the CP violating Dirac phase to be $\pm(0.09\pi \sim 0.76\pi)$ for NH ($\pm(0.15\pi \sim 0.73\pi)$ for IH) depending on $\sin^2 \theta_{23}$. The rotation of the 1-3 generations of neutrinos in the TBM basis gives $\sin^2 \theta_{12} > 1/3$. The CP violating Dirac phase is not constrained by the input of the present experimental data. For the case of the 1-3 and 1-2 rotations of charged leptons in the TBM basis, the CP violating Dirac phase is predicted in $\pm(0.35\pi \sim 0.60\pi)$ depending on $\sin^2 \theta_{12}$ for both NH and IH cases. We also discuss the specific case that θ_{13} is related with the Cabibbo angle λ such as $\sin \theta_{13} = \lambda/\sqrt{2}$, in which the maximal CP violation is preferred. The CP violating Dirac phase can distinguish the lepton flavor mixing patterns at T2K and NO ν A experiments in the future.

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1 Introduction

The neutrino oscillation experiments have been revealed the neutrino masses and the two large neutrino mixing angles. Reactor neutrino experiments have also observed a non-zero θ_{13} , which is the last mixing angle of the lepton sector [1, 2, 3, 4]. The T2K experiment has confirmed the neutrino oscillation in the $\nu_\mu \rightarrow \nu_e$ appearance events [5, 6], which provide us the new information of the CP violation of the lepton sector by combining the data of reactor experiments. Thus, recent neutrino oscillation experiments go into a new phase of precise determinations of the lepton mixing matrix and neutrino mass squared differences [7, 8, 9]. Therefore, the detailed study of the neutrino mixing including the CP violating phase gives us clues to reach the flavor theory.

Before the reactor experiments reported the non-zero value of θ_{13} , there was a paradigm of "tri-bimaximal mixing" (TBM) [10, 11], which is a simple mixing pattern for leptons and can be easily derived from flavor symmetries. At early stage some authors succeeded to realize the TBM in the A_4 models [12, 13, 14, 15, 16]. Then, the non-Abelian discrete groups are center of attention at the flavor symmetry [17, 18, 19]. The other groups were also considered to give the TBM [20]-[28]. On the other hand, the deviation from the TBM was also discussed [29, 30] and the magnitudes of θ_{13} was estimated based on models [31]-[45]. The observation of the non-vanishing θ_{13} forces to study the deviation from the TBM [46]-[53] precisely or study other flavor paradigms, e.g. tri-bimaximal-Cabibbo mixing [54, 55].

It is the important problem for the flavor physics whether the TBM basis has a physical background such as the flavor symmetry or not. In order to answer this question, one should examine the flavor mixing angles and the CP violating Dirac phase strictly [56, 57, 58, 59, 60]. In this work, we discuss predictions in the typical four cases of the deviation from the TBM, and propose to test the CP violating Dirac phase at the future experiments, T2K and NO ν A. The first two cases are given by the additional rotation of 2-3 or 1-3 generations of neutrinos in the TBM basis, which is called as the tri-maximal mixing [61]. Other two cases are given by the additional rotation of the 1-3 or 1-2 generations of charged leptons in the TBM basis, which are not any more the tri-maximal mixing. These four cases give different relations among three mixing angles and the CP violating Dirac phase.

First, we discuss the rotation of the 2-3 generations of neutrinos in the TBM basis, which predicts $\sin^2 \theta_{12} < 1/3$. It is also found that the CP violating Dirac phase δ_{CP} is non-zero and depends on the magnitude of $\sin^2 \theta_{23}$. The case of the rotation of the 1-3 generations of neutrinos in the TBM basis gives $\sin^2 \theta_{12} > 1/3$. The CP violating Dirac phase is not constrained by the input of the present experimental data. For the case of the rotation of the 1-3 and 2-3 generations of charged leptons in the TBM basis, $\sin^2 \theta_{23}$ is very close to $1/2$ with larger than $1/2$ (1-3 rotation) or smaller than $1/2$ (1-2 rotation). For both cases, the CP violating Dirac phase is predicted to be in the narrow range of $\pm(0.35\pi \sim 0.60\pi)$, which may be preferred by the recent T2K experiment data [5, 6]. Thus, the CP violating Dirac phase can distinguish the lepton flavor mixing patterns deviating from the TBM.

In addition, we discuss the specific case where the θ_{13} is related with the Cabibbo angle. In the framework of Grand Unified Theories (GUT), where the Yukawa matrices for the charged leptons and the down-type quarks have the same origin, the 1-2 generation mixing angle of the charged leptons is same as the Cabibbo angle λ [54, 62]. Therefore, one obtains

$\sin \theta_{13} = \lambda/\sqrt{2}$, which is in excellent agreement with the observed value. Then, we predict the magnitude of the CP violating phase.

The paper is organized as follows. We discuss the non-trivial relations among mixing angles and the CP violating Dirac phase in the four cases of the deviation from the TBM in the section 2. In the section 3, we present numerical predictions. The section 4 is devoted to the summary.

2 Deviation from tri-bimaximal mixing

Let us discuss the structure of the lepton mixing matrix, so called the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix U_{PMNS} [63, 64]. Three mixing angles θ_{ij} ($i, j = 1, 2, 3$; $i < j$) and one CP violating Dirac phase δ_{CP} are parameterized as the PDG form [65]:

$$U_{\text{PMNS}} \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix}, \quad (1)$$

where c_{ij} and s_{ij} denote $\cos \theta_{ij}$ and $\sin \theta_{ij}$, respectively. If neutrinos are Majorana particles, Majorana phases are included in the left-handed Majorana neutrino masses. As well known the Jarlskog invariant [66], which is the parameter describing the size of the CP violation, is written as

$$J_{CP} = \text{Im} [U_{e1}U_{\mu 2}U_{e2}^*U_{\mu 1}^*], \quad (2)$$

where U_{ij} s are the PMNS matrix elements. Then, the J_{CP} is written in terms of the lepton mixing angles and the CP violating Dirac phase as

$$J_{CP} = s_{23}c_{23}s_{12}c_{12}s_{13}c_{13}^2 \sin \delta_{CP}. \quad (3)$$

For the lepton mixing matrix, Harrison-Perkins-Scott proposed a simple form of the mixing pattern, so-called the tri-bimaximal mixing [10, 11] as follows:

$$V_{\text{TBM}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (4)$$

Before the reactor experiments reported non-zero θ_{13} [1, 2, 3, 4], the TBM was good scheme for the lepton sector. In our work, we assume the TBM to be a good starting point of the lepton mixing. Then, we discuss the deviation from the TBM, which originates in the neutrino sector or in the charged lepton sector. These different origins give us the different flavor mixing patterns.

2.1 Deviation in the neutrino sector

First of all, we consider an additional rotation of 2-3 generations of neutrinos in the TBM basis [50, 55], which is called as Case I. In order to obtain the PMNS mixing matrix, we

multiply the TBM matrix V_{TBM} by the 2-3 rotation matrix in the right-hand side as follows:

$$U_{\text{PMNS}} = V_{\text{TBM}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \phi & e^{-i\sigma} \sin \phi \\ 0 & -e^{i\sigma} \sin \phi & \cos \phi \end{pmatrix}, \quad (5)$$

where ϕ and σ are arbitrary parameters, which are determined by the experimental data. Then, the relevant mixing matrix elements are given as

$$|U_{e2}| = \left| \frac{\cos \phi}{\sqrt{3}} \right|, \quad |U_{e3}| = \left| \frac{e^{-i\sigma} \sin \phi}{\sqrt{3}} \right|, \quad |U_{\mu 3}| = \left| -\frac{\cos \phi}{\sqrt{2}} + \frac{e^{-i\sigma} \sin \phi}{\sqrt{3}} \right|, \quad (6)$$

which are converted to

$$\sin^2 \theta_{12} = 1 - \frac{2}{3 - \sin^2 \phi}, \quad \sin^2 \theta_{13} = \frac{1}{3} \sin^2 \phi, \quad \sin^2 \theta_{23} = \frac{1}{2} \left(1 - \frac{\sqrt{6} \sin 2\phi \cos \sigma}{3 - \sin^2 \phi} \right). \quad (7)$$

The Jarlskog invariant J_{CP} is given as

$$J_{CP} = s_{23} c_{23} s_{12} c_{12} s_{13} c_{13}^2 \sin \delta_{CP} = -\frac{1}{6\sqrt{6}} \sin 2\phi \sin \sigma, \quad (8)$$

and then, the CP phase δ_{CP} is expressed in terms of two parameters ϕ and σ as follows:

$$\sin \delta_{CP} = -\frac{(5 + \cos 2\phi) \sin \sigma}{\sqrt{(5 + \cos 2\phi)^2 - 24 \sin^2 2\phi \cos^2 \sigma}}. \quad (9)$$

Since this case does not change the first column of the TBM mixing matrix, this is called as TM1, which is one of the tri-maximal mixing patterns. Then, $\cos \delta_{CP}$ is easily obtained by putting $|U_{\mu 1}|^2 = 1/6$. Now, we can write down the sum rules among the mixing angles and the phase δ_{CP} from Eqs. (7) and (9) [39]:

$$\sin^2 \theta_{12} = 1 - \frac{2}{3 \cos^2 \theta_{13}} \leq \frac{1}{3}, \quad \cos \delta_{CP} \tan 2\theta_{23} \simeq -\frac{1}{2\sqrt{2} \sin \theta_{13}} \left(1 - \frac{7}{2} \sin^2 \theta_{13} \right), \quad (10)$$

which are helpful to understand our numerical results in the section 3.

It is remarked that this mixing pattern is derived from the neutrino mass matrix $m_{\nu LL}$ of the A_4 and S_4 models [50, 67]:

$$m_{\nu LL} = a \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + b \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + d \begin{pmatrix} 0 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & -2 \end{pmatrix}, \quad (11)$$

where a , b , c and d are arbitrary complex parameters.

Next, we discuss Case II, in which an additional rotation of 1-3 generations of neutrinos is taken in the TBM basis [29, 40, 43]. The PMNS matrix is given as

$$U_{\text{PMNS}} = V_{\text{TBM}} \begin{pmatrix} \cos \phi & 0 & e^{-i\sigma} \sin \phi \\ 0 & 1 & 0 \\ -e^{i\sigma} \sin \phi & 0 & \cos \phi \end{pmatrix}. \quad (12)$$

The relevant mixing matrix elements are written as

$$|U_{e2}| = \frac{1}{\sqrt{3}}, \quad |U_{e3}| = \left| \frac{2e^{-i\sigma} \sin \phi}{\sqrt{6}} \right|, \quad |U_{\mu 3}| = \left| -\frac{\cos \phi}{\sqrt{2}} - \frac{e^{-i\sigma} \sin \phi}{\sqrt{6}} \right|, \quad (13)$$

which are converted to

$$\sin^2 \theta_{12} = \frac{1}{3 - 2 \sin^2 \phi}, \quad \sin^2 \theta_{13} = \frac{2}{3} \sin^2 \phi, \quad \sin^2 \theta_{23} = \frac{1}{2} \left(1 + \frac{\sqrt{3} \sin 2\phi \cos \sigma}{3 - 2 \sin^2 \phi} \right). \quad (14)$$

The Jarlskog invariant J_{CP} is given as follows:

$$J_{CP} = -\frac{1}{6\sqrt{3}} \sin 2\phi \sin \sigma. \quad (15)$$

Therefore, the CP phase δ_{CP} is given as

$$\sin \delta_{CP} = -\frac{(2 + \cos 2\phi) \sin \sigma}{\sqrt{(2 + \cos 2\phi)^2 - 3 \sin^2 2\phi \cos^2 \sigma}}. \quad (16)$$

The rotation of the 1-3 generations of neutrinos in the TBM basis gives another tri-maximal mixing pattern, in which the second column of the TBM mixing matrix is not changed, so called TM2. Then, $\cos \delta_{CP}$ is obtained by putting $|U_{\mu 2}|^2 = 1/3$. The sum rules among the mixing angles and the phase δ_{CP} are given by from Eqs. (14) and (16) [39]:

$$\sin^2 \theta_{12} = \frac{1}{3} \frac{1}{\cos^2 \theta_{13}} \geq \frac{1}{3}, \quad \cos \delta_{CP} \tan 2\theta_{23} \simeq \frac{1}{\sqrt{2} \sin \theta_{13}} \left(1 - \frac{5}{4} \sin^2 \theta_{13} \right). \quad (17)$$

This mixing pattern is obtained in the neutrino mass matrix $m_{\nu LL}$ of the A_4 and S_4 models [40, 43, 52, 53, 68]:

$$m_{\nu LL} = a \begin{pmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{pmatrix} + b \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + c \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix} + d \begin{pmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, \quad (18)$$

where a , b , c and d are arbitrary complex parameters.

Since the rotation of 1-2 generations leads to $\theta_{13} = 0$, we do not discuss this case.

2.2 Deviation in the charged lepton sector

The TBM is realized in the specific flavor structure of the neutrino mass matrix in the basis of the diagonal charged lepton mass matrix, which is given in some flavor symmetries. If an additional flavor symmetry breaking term causes the deviation from the diagonal charged lepton mass matrix, the lepton mixing matrix is not anymore the tri-maximal mixing.

Let us discuss Case III, in which the neutrino mixing is the TBM one, on the other hand, the diagonal charged lepton mass matrix is rotated between 1-3 generations. In order to

obtain the PMNS mixing matrix, we multiply the TBM matrix by the 1-3 rotation matrix in the left-hand side as follows:

$$U_{\text{PMNS}} = \begin{pmatrix} \cos \phi & 0 & -e^{-i\sigma} \sin \phi \\ 0 & 1 & 0 \\ e^{i\sigma} \sin \phi & 0 & \cos \phi \end{pmatrix} V_{\text{TBM}} . \quad (19)$$

Then, the relevant mixing matrix elements are given as

$$|U_{e2}| = \left| \frac{\cos \phi}{\sqrt{3}} - \frac{e^{-i\sigma} \sin \phi}{\sqrt{3}} \right|, \quad |U_{e3}| = \left| -\frac{e^{-i\sigma} \sin \phi}{\sqrt{2}} \right|, \quad |U_{\mu 3}| = \frac{1}{\sqrt{2}}, \quad (20)$$

which are converted to

$$\sin^2 \theta_{12} = \frac{2(1 - \sin 2\phi \cos \sigma)}{3(2 - \sin^2 \phi)}, \quad \sin^2 \theta_{13} = \frac{1}{2} \sin^2 \phi, \quad \sin^2 \theta_{23} = \frac{1}{2 - \sin^2 \phi}. \quad (21)$$

The Jarlskog invariant J_{CP} is given as

$$J_{CP} = \frac{1}{12} \sin 2\phi \sin \sigma. \quad (22)$$

Therefore, the CP phase δ_{CP} is written as

$$\sin \delta_{CP} = \frac{(2 - \sin^2 \phi) \sin \sigma}{\sqrt{(4 - 3 \sin^2 \phi + 2 \sin 2\phi \cos \sigma)(1 - \sin 2\phi \cos \sigma)}}. \quad (23)$$

On the other hand, $\cos \delta_{CP}$ is obtained by putting $|U_{\mu 1}|^2 = 1/6$ as well as in Case I. The sum rules among the mixing angles and the phase δ_{CP} are given from Eqs. (21) and (23) [39]:

$$\sin^2 \theta_{23} = \frac{1}{2} \frac{1}{\cos^2 \theta_{13}} \geq \frac{1}{2}, \quad \sin^2 \theta_{12} \simeq \frac{1}{3} - \frac{2\sqrt{2}}{3} \sin \theta_{13} \cos \delta_{CP} + \frac{1}{3} \sin^2 \theta_{13} \cos 2\delta_{CP}. \quad (24)$$

Finally, we discuss Case IV, in which the neutrino mixing is the TBM one, on the other hand, the diagonal charged lepton mass matrix is rotated between 1-2 generations. The PMNS mixing matrix is given as

$$U_{\text{PMNS}} = \begin{pmatrix} \cos \phi & -e^{-i\sigma} \sin \phi & 0 \\ e^{i\sigma} \sin \phi & \cos \phi & 0 \\ 0 & 0 & 1 \end{pmatrix} V_{\text{TBM}}. \quad (25)$$

The relevant mixing matrix elements are

$$|U_{e2}| = \left| \frac{\cos \phi}{\sqrt{3}} - \frac{e^{-i\sigma} \sin \phi}{\sqrt{3}} \right|, \quad |U_{e3}| = \left| \frac{e^{-i\sigma} \sin \phi}{\sqrt{2}} \right|, \quad |U_{\mu 3}| = \left| -\frac{\cos \phi}{\sqrt{2}} \right|, \quad (26)$$

which are converted to

$$\sin^2 \theta_{12} = \frac{2(1 - \sin 2\phi \cos \sigma)}{3(2 - \sin^2 \phi)}, \quad \sin^2 \theta_{13} = \frac{1}{2} \sin^2 \phi, \quad \sin^2 \theta_{23} = 1 - \frac{1}{2 - \sin^2 \phi}. \quad (27)$$

The Jarlskog invariant J_{CP} is given as follows:

$$J_{CP} = -\frac{1}{12} \sin 2\phi \sin \sigma . \quad (28)$$

The CP phase δ_{CP} is

$$\sin \delta_{CP} = -\frac{(2 - \sin^2 \phi) \sin \sigma}{\sqrt{(4 - 3 \sin^2 \phi + 2 \sin 2\phi \cos \sigma)(1 - \sin 2\phi \cos \sigma)}} , \quad (29)$$

and $\cos \delta_{CP}$ is given by putting $|U_{\tau 1}|^2 = 1/6$.

It is noticed that the mixing angles in Eq.(27) are obtained by replacing $\sin^2 \theta_{23}$ with $\cos^2 \theta_{23}$ in Eq.(21), and $\sin \delta_{CP}$ is the same as the one of Case III except \pm sign. The sum rules among the mixing angles and the phase δ_{CP} are given by from Eqs.(26) and (29) [39]:

$$\sin^2 \theta_{23} = 1 - \frac{1}{2 \cos^2 \theta_{13}} \leq \frac{1}{2}, \quad \sin^2 \theta_{12} \simeq \frac{1}{3} + \frac{2\sqrt{2}}{3} \sin \theta_{13} \cos \delta_{CP} + \frac{1}{3} \sin^2 \theta_{13} \cos 2\delta_{CP}. \quad (30)$$

We also discuss the specific case in Case IV, where θ_{13} is related with the Cabibbo angle λ . In the framework of GUT, where the Yukawa matrices for the charged leptons and the down-type quarks have the same origin, the 1-2 generation mixing angle $\sin \phi$ is the same as the Cabibbo angle λ [54, 62]. Therefore, one obtains $\sin \theta_{13} = \lambda/\sqrt{2} \simeq 0.16$, which is in excellent agreement with the observed value. Then, the magnitude of the CP violating phase is predicted to be in the narrow range.

Since the rotation of 2-3 generations gives $\theta_{13} = 0$, we do not discuss this case. By using above formulas, the numerical studies are discussed in the next section.

3 Numerical analysis

As input data in our calculations, we use the results of the global analysis of the neutrino oscillation experiments for three mixing angles [7]. For the case of the normal hierarchy (NH) of neutrino masses, we take

$$\sin^2 \theta_{12} = 0.320_{-0.017}^{+0.016}, \quad \sin^2 \theta_{13} = 0.0246_{-0.0028}^{+0.0029}, \quad \sin^2 \theta_{23} = 0.613_{-0.040}^{+0.022} (0.427_{-0.027}^{+0.034}), \quad (31)$$

at 1 σ level, where there are two solutions for $\sin^2 \theta_{23}$, and

$$0.27 < \sin^2 \theta_{12} < 0.37, \quad 0.017 < \sin^2 \theta_{13} < 0.033, \quad 0.36 < \sin^2 \theta_{23} < 0.68, \quad (32)$$

at 3 σ level. For the case of the inverted hierarchy (IH) of neutrino masses, we take

$$\sin^2 \theta_{12} = 0.320_{-0.017}^{+0.016}, \quad \sin^2 \theta_{13} = 0.0250_{-0.0027}^{+0.0026}, \quad \sin^2 \theta_{23} = 0.600_{-0.031}^{+0.026}, \quad (33)$$

at 1 σ level. We have at 3 σ level:

$$0.27 < \sin^2 \theta_{12} < 0.37, \quad 0.017 < \sin^2 \theta_{13} < 0.033, \quad 0.37 < \sin^2 \theta_{23} < 0.67. \quad (34)$$

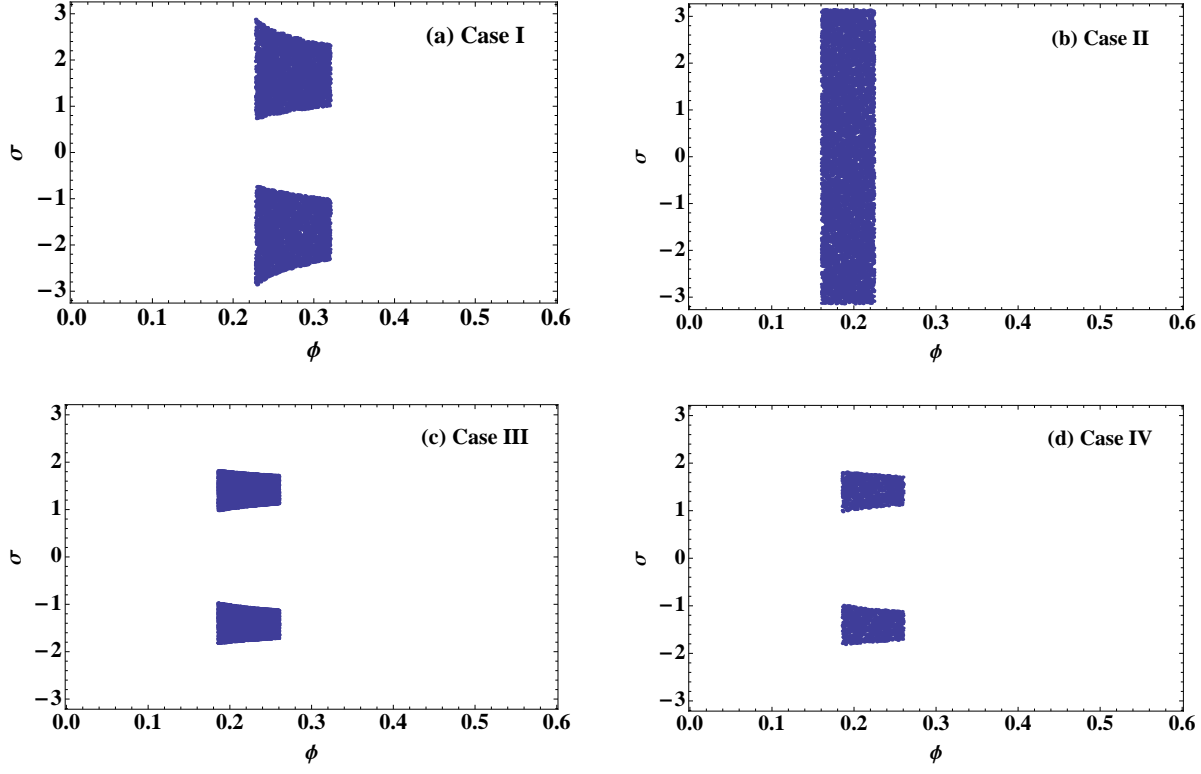


Figure 1: Allowed regions on ϕ - σ plane for NH in (a) Case I, (b) Case II, (c) Case III and (d) Case IV, where the experimental data are input at 3σ level.

Our numerical procedure is as follows. The free parameters ϕ and σ are scattered in the regions of $0 < \phi < \pi/2$ and $-\pi < \sigma < \pi$, respectively, and then, we obtain the regions of ϕ and σ which are allowed by the experimental data of three mixing angles θ_{12} , θ_{23} and θ_{13} at 3σ level. Using these allowed sets ϕ and σ , we predict the CP violating phase δ_{CP} by plugging back to, for example, Eqs. (7) and (9) for Case I. We show the allowed regions of ϕ and σ for the NH case in Fig. 1. The allowed region in Case III and Case IV are just same because the region of ϕ is determined practically only by the experimental data of $\sin^2 \theta_{13}$, which is the same form as seen in Eqs.(21) and (27), and $\sin \delta_{CP}$ is the same one except the \pm sign as seen in Eqs. (23) and (29).

The CP violating phase δ_{CP} is predicted by these allowed region in Fig.1. By using these allowed sets (ϕ, σ) , we show relations among the mixing angles and the CP violating phase numerically.

3.1 Case I

Let us discuss the NH case, where the data of Eq.(32) are input. The IH case is only different from the NH one in the input data of $\sin^2 \theta_{23}$. We start with presenting the result for the Case I, in which the 2-3 plane of neutrino generations is rotated in the TBM basis as in Eq.(5).

We show the prediction on the $\sin^2 \theta_{12}$ – $\sin^2 \theta_{13}$ plane in Fig. 2. It is found that the predicted linear relation between $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$ is completely consistent with the experimental data. We obtain $\sin^2 \theta_{12} \leq 1/3$ as seen in the sum rule of Eq.(10). Actually, the allowed region of $\sin^2 \theta_{12}$ is restricted by the observed value of $\sin^2 \theta_{13}$ such as $0.310 < \sin^2 \theta_{12} < 0.323$. On the other hand, since there is no clear correlation between $\sin^2 \theta_{23}$ and $\sin^2 \theta_{13}$ as seen in Eq.(7), we omit it in our figures.

Recently, the T2K experiment has presented new data [6], in which the CP violating Dirac phase is $-1.18\pi < \delta_{CP} < 0.15\pi$ ($-0.91\pi < \delta_{CP} < -0.08\pi$) for the NH (IH) with $\sin^2 \theta_{23} = 0.514^{+0.055}_{-0.056}$ (0.511 ± 0.055). We show the prediction for the CP violating Dirac phase versus $\sin^2 2\theta_{13}$ in Fig. 3. The CP violating Dirac phase is predicted as $0.09\pi \lesssim |\delta_{CP}| \lesssim 0.76\pi$,

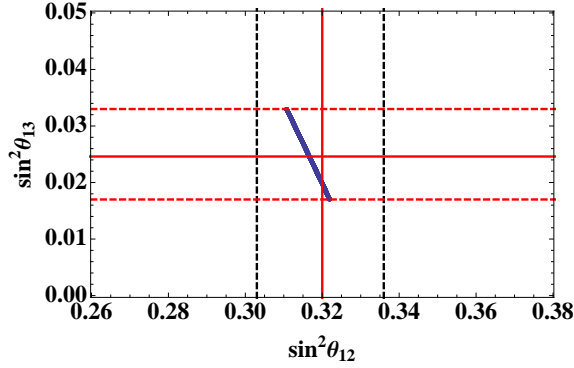


Figure 2: The prediction on the $\sin^2 \theta_{12}$ – $\sin^2 \theta_{13}$ plane for NH in Case I. The red solid, black dashed, and red dashed lines denote the experimental best fit, 1σ and 3σ level for NH, respectively.

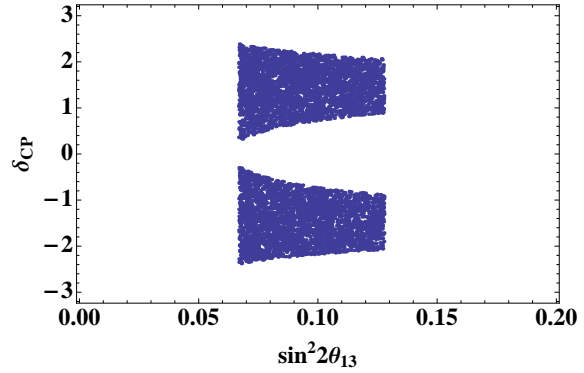


Figure 3: The prediction on the $\sin^2 2\theta_{13}$ – δ_{CP} plane for NH in Case I, where $\sin^2 2\theta_{13}$ is cut by the experimental data.

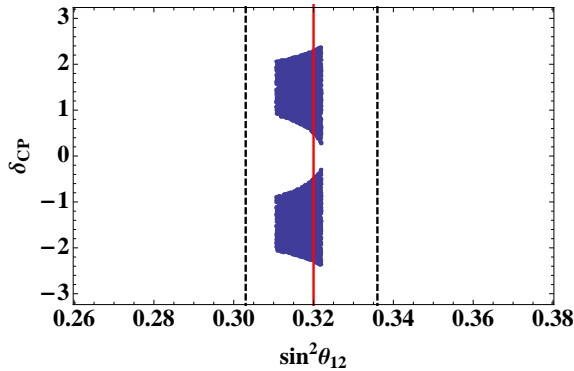


Figure 4: The prediction on the $\sin^2 \theta_{12}$ – δ_{CP} plane for NH in Case I. The red solid and black dashed lines denote the experimental best fit and 1σ level for NH, respectively.

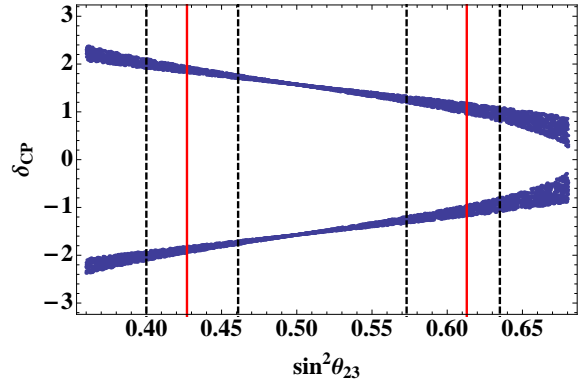


Figure 5: The prediction on the $\sin^2 \theta_{23}$ – δ_{CP} plane for NH in Case I. The two red solid lines and the black dashed lines denote the experimental best fits and at 1σ level, respectively, for NH.

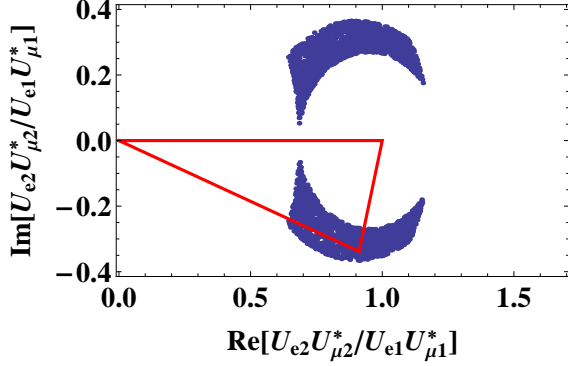


Figure 6: The predicted vertex in the unitarity triangle for NH in Case I. The red triangle denotes a reference one.

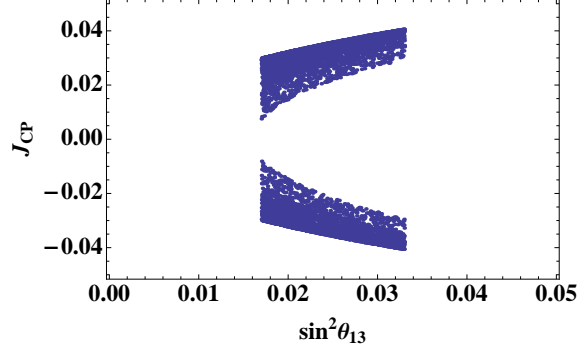


Figure 7: The predicted plot of J_{CP} versus $\sin^2 \theta_{13}$ for NH in Case I.

which will be tested in the future.

In order to see other mixing angle dependences of δ_{CP} , we show δ_{CP} versus $\sin^2 \theta_{12}$ and $\sin^2 \theta_{23}$ in Figs. 4 and 5, respectively. As seen in the sum rules of Eq.(10), our predicted δ_{CP} depends on $\sin^2 \theta_{23}$ clearly, but does not so on $\sin^2 \theta_{12}$. The clear dependence between δ_{CP} and $\sin^2 \theta_{23}$ is attributed to the precise measurement of $\sin \theta_{13}$ as seen in Eq.(10). The more precise data of $\sin \theta_{13}$ will be helpful to test this case.

It is found that δ_{CP} is maximal $\pm\pi/2$ at $\sin^2 \theta_{23} = 1/2$. If $\sin^2 \theta_{23}$ is smaller (larger) than $1/2$, $|\delta_{CP}|$ is larger (smaller) than $\pi/2$. Thus, we can test our prediction clearly since the observed value of $\sin^2 \theta_{23}$ is improved in the future experiments.

For the IH case, the predicted lower bound is reduced to $0.15\pi \lesssim |\delta_{CP}| \lesssim 0.73\pi$ since the allowed region of $\sin^2 \theta_{23}$ is narrow slightly compared with the NH case.

Let us discuss the unitarity triangle of leptons. The unitarity condition

$$U_{e1}U_{\mu 1}^* + U_{e2}U_{\mu 2}^* + U_{e3}U_{\mu 3}^* = 0 \quad (35)$$

is expressed as a triangle on the complex plane. We show the predicted vertex of the triangle in Fig. 6, in which the reference triangle at

$$\delta_{CP} = -\frac{\pi}{2}, \quad \sin^2 \theta_{13} = 0.0251, \quad \sin^2 \theta_{12} = 0.312, \quad \sin^2 \theta_{23} = 0.514, \quad (36)$$

is presented for the eye guide. Thus, the flavor mixing will be tested in the unitarity triangle. We also show J_{CP} versus $\sin^2 \theta_{13}$ in Fig. 7. The predicted region is $0.005 < |J_{CP}| < 0.04$.

3.2 Case II

We discuss the Case II, in which the 1-3 plane of neutrino generations is rotated in the TBM basis as in Eq.(12). We show the predicted line on the $\sin^2 \theta_{12}$ – $\sin^2 \theta_{13}$ plane in Fig. 8. In this case, the predicted $\sin^2 \theta_{12}$ is outside of 1σ of the experimental data, which is contrast with the one in the Case I. We obtain $\sin^2 \theta_{12} \geq 1/3$ as seen in the sum rule of Eq.(17).

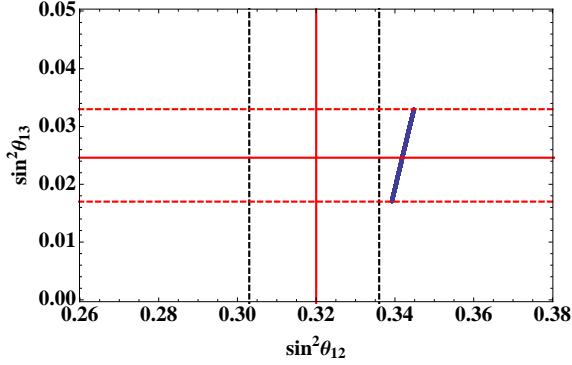


Figure 8: The prediction on the $\sin^2 \theta_{12}$ – $\sin^2 \theta_{13}$ plane for NH in Case II. The red solid, black dashed, and red dashed lines denote the experimental best fit, 1σ and 3σ level for NH, respectively.

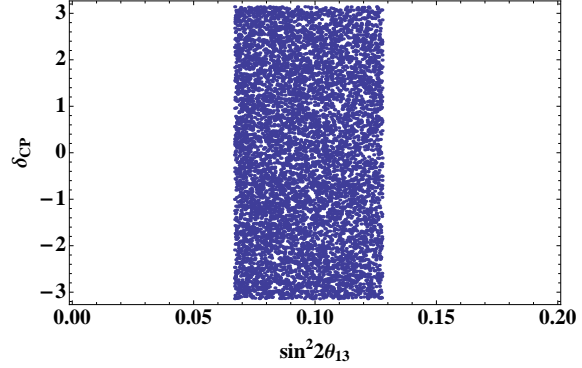


Figure 9: The prediction on the $\sin^2 2\theta_{13}$ – δ_{CP} plane for NH in Case II, where $\sin^2 2\theta_{13}$ is cut by the experimental data.

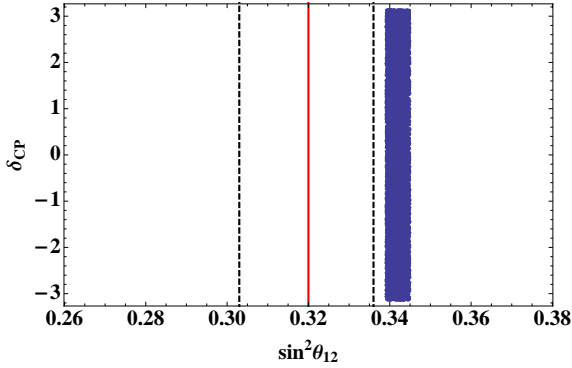


Figure 10: The allowed region on the $\sin^2 \theta_{12}$ – δ_{CP} plane for NH in Case II. The red solid and black dashed lines denote the experimental best fit and at 1σ level for NH, respectively.

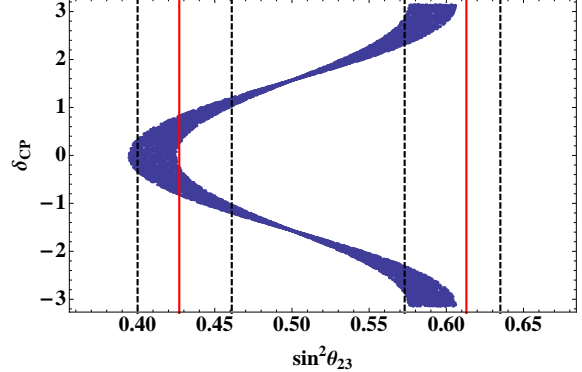


Figure 11: The allowed region on the $\sin^2 \theta_{23}$ – δ_{CP} plane for NH in Case II. The two red solid lines and the black dashed lines denote the experimental best fits and at 1σ level, respectively, for NH.

We show the predicted δ_{CP} versus $\sin^2 2\theta_{13}$ in Fig. 9. The predicted δ_{CP} is allowed in all region of $-\pi \sim \pi$. In order to see other mixing angle dependences of δ_{CP} , we show the predicted δ_{CP} versus $\sin^2 \theta_{12}$ and $\sin^2 \theta_{23}$ in Figs. 10 and 11, respectively. There is no $\sin^2 \theta_{12}$ dependence for the predicted δ_{CP} as seen in Eq.(14). On the other hand, there is a remarkable $\sin^2 \theta_{23}$ dependence for δ_{CP} due to the precise data of $\sin \theta_{13}$ as seen in the sum rule of Eq.(17). It is noticed that δ_{CP} is maximal, $\pm\pi/2$ at $\sin^2 \theta_{23} = 1/2$ as well as in Case I. If $\sin^2 \theta_{23}$ is smaller (larger) than $1/2$, $|\delta_{CP}|$ is smaller (larger) than $\pi/2$. Thus, we can test our prediction in the future since the observed value of $\sin^2 \theta_{23}$ is improved in the future experiments as well as in Case I. It is also noted that the $\sin^2 \theta_{23}$ is predicted as $0.39 \lesssim \sin^2 \theta_{23} \lesssim 0.61$, which is within the experimental bound of 3σ level, $0.36 < \sin^2 \theta_{23} < 0.68$.

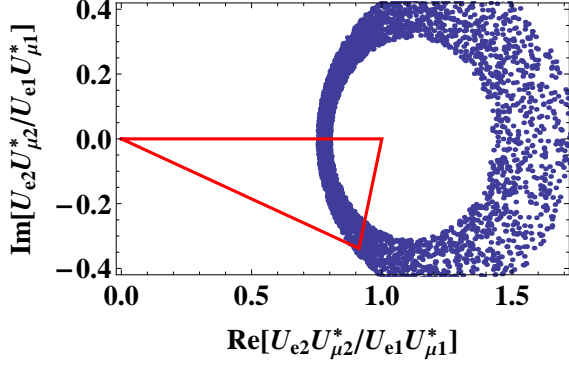


Figure 12: The predicted vertex in the unitarity triangle with a reference one for NH in Case II.

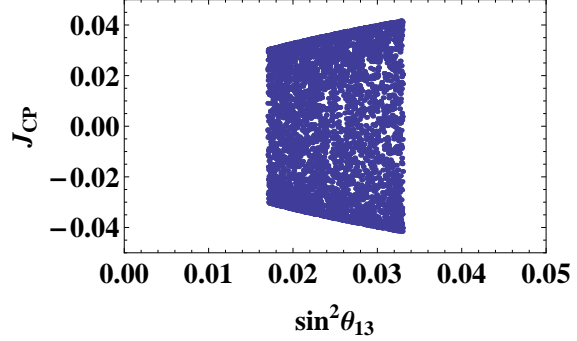


Figure 13: The predicted plot of J_{CP} versus $\sin^2 \theta_{13}$ for NH in Case II.

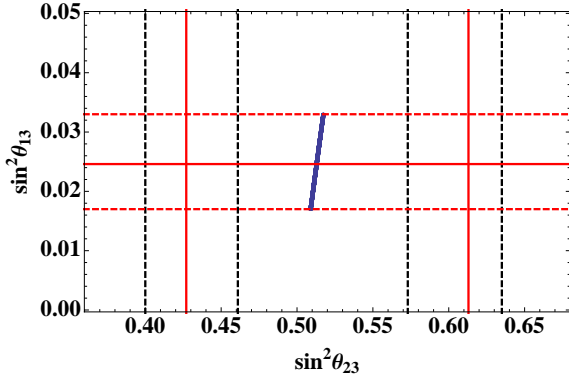


Figure 14: The prediction on the $\sin^2 \theta_{12}$ – $\sin^2 \theta_{13}$ plane for NH in Case III. The red solid, black dashed, and red dashed lines denote the experimental best fit, 1σ and 3σ level for NH, respectively.

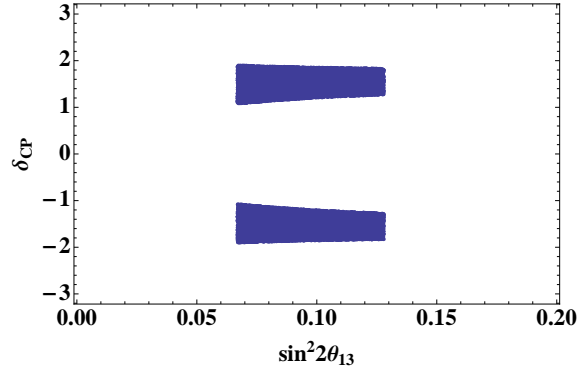


Figure 15: The prediction on the $\sin^2 2\theta_{13}$ – δ_{CP} plane for NH in Case III, where $\sin^2 2\theta_{13}$ is cut by the experimental data.

In Fig. 12, we show the vertex in the unitarity triangle, which is still allowed in the wide region. We also show the Jarlskog invariant J_{CP} versus $\sin^2 \theta_{13}$ in Fig. 13. The J_{CP} is predicted to be in the region of $-0.04 \lesssim J_{CP} \lesssim 0.04$.

For the IH case, the predictions are the same as the ones in the NH case, because the predicted region of $\sin^2 \theta_{23}$ is inside of the experimental data of 3σ as seen in Fig.11.

3.3 Case III

We discuss Case III, in which the 1-3 plane of charged lepton generations is rotated as in Eq.(19). We show the prediction on the $\sin^2 \theta_{23}$ – $\sin^2 \theta_{13}$ plane in Fig. 14. The predicted $\sin^2 \theta_{23}$ is outside of 1σ of the experimental data. The predicted $\sin^2 \theta_{23}$ is slightly larger than $1/2$ as seen in the sum rule of Eq.(24). The predicted $\sin^2 \theta_{13}$ is sensitive to the magnitude

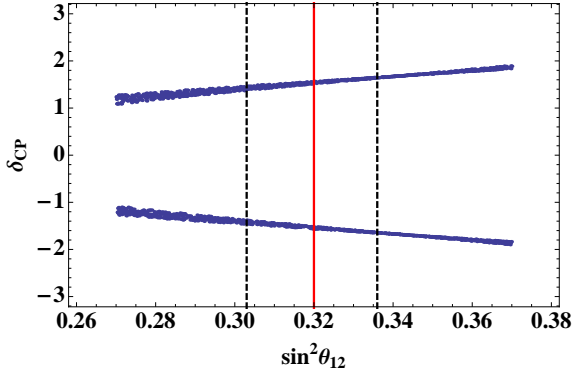


Figure 16: The allowed region on the $\sin^2 \theta_{12}$ - δ_{CP} plane for NH in Case III. The red solid and black dashed lines denote the experimental best fit and at 1σ level for NH, respectively.

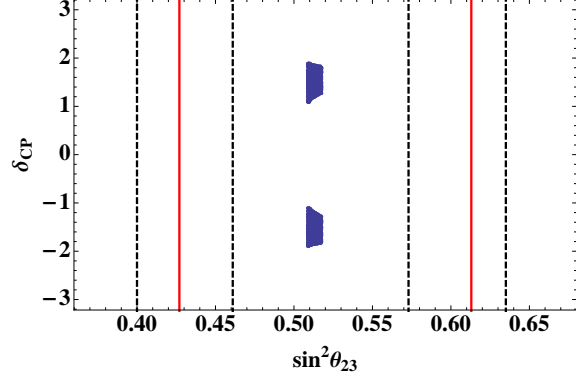


Figure 17: The allowed region on the $\sin^2 \theta_{23}$ - δ_{CP} plane for NH in Case III. The two red solid lines and the black dashed lines denote the experimental best fits and at 1σ level, respectively, for NH.

of $\sin^2 \theta_{23}$. On the other hand, there is no correlation between $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$. This is contrast to Case I and Case II. The figure is omitted in this paper.

We show the scatter plot of δ_{CP} versus $\sin^2 2\theta_{13}$ in Fig. 15. The δ_{CP} is predicted to be $\delta_{CP} \simeq \pm(0.35\pi \sim 0.60\pi)$, which is testable in the future experiments. We show δ_{CP} versus $\sin^2 \theta_{12}$ and $\sin^2 \theta_{23}$ in Figs. 16 and 17, respectively. There is a clear $\sin^2 \theta_{12}$ dependence for δ_{CP} due to the precise data of $\sin \theta_{13}$, which is understood in the sum rule of Eq.(24). It is remarked that δ_{CP} is maximal value $\pm\pi/2$ at $\sin^2 \theta_{12} \simeq 1/3$. If $\sin^2 \theta_{12}$ is smaller (larger) than $1/3$, $|\delta_{CP}|$ is smaller (larger) than $\pi/2$. Thus, we can test our prediction by the precise data of $\sin^2 \theta_{12}$ in the future experiments.

In Fig. 18, we show the unitarity triangle. The predicted region of the vertex is narrow compared with the ones in Case I and Case II. We also show the Jarlskog invariant J_{CP} versus $\sin^2 \theta_{13}$ in Fig. 19. The J_{CP} is predicted to be in the narrow region of J_{CP} as $0.025 \lesssim |J_{CP}| \lesssim 0.04$.

For the IH case, the predictions are the same as the one in the NH case as well as the Case II, because the predicted region of $\sin^2 \theta_{23}$ is near the maximal one as seen in Fig. 16.

3.4 Case IV

For Case IV, in which the 1-2 plane of the charged lepton generations is rotated as in Eq.(25), we obtain the similar results in the Case III. The predictions are obtained by replacing $\sin^2 \theta_{23}$ with $\cos^2 \theta_{23}$ in Case III. This situation is easily understood by comparing with Eqs.(21) and (27), or the sum rules Eqs.(24) and (30). Therefore, we omit figures of numerical results.

Instead, we add some comments as follows. The predicted $\sin^2 \theta_{23}$ is slightly smaller than $1/2$ as seen in the sum rule of Eq.(30). The CP violating Dirac phase δ_{CP} depends on $\sin^2 \theta_{12}$ as well as in Case III, however if $\sin^2 \theta_{12}$ is smaller (larger) than $1/3$, $|\delta_{CP}|$ is larger (smaller) than $\pi/2$, which is contrast to the one in Fig. 16. Thus, the precise data of $\sin^2 \theta_{12}$ can distinguish Case III and Case IV in the future.

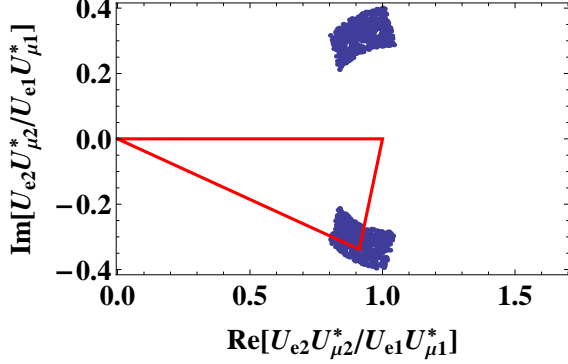


Figure 18: The predicted vertex in the unitarity triangle for NH in Case III.

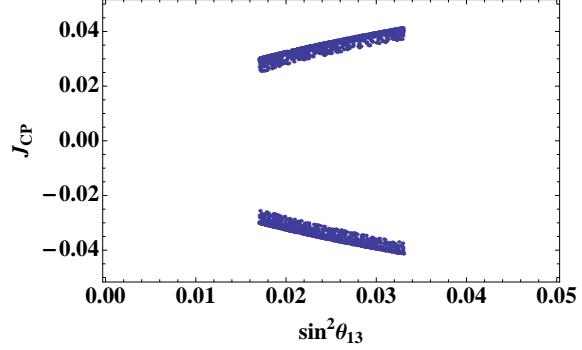


Figure 19: The predicted plot of J_{CP} versus $\sin^2 \theta_{13}$ for NH in Case III.

Finally, we discuss the specific case of Case IV, in which θ_{13} is related with the Cabibbo angle such as $\sin \theta_{13} = \lambda/\sqrt{2} \simeq 0.16$. This relation can be derived from the framework of GUT, where the Yukawa matrices for the charged leptons and the down-type quarks have the same origin. We obtain the magnitude of the CP violating phase to be in the narrow region $\pm(0.41\pi \sim 0.62\pi)$, which may prefer the maximal CP violation, $\delta_{CP} = \pm\pi/2$.

4 Summary

We examine non-trivial relations among the mixing angles and the CP violating Dirac phase in the typical four cases of the deviation from the TBM. The first two cases are given by the additional rotation of the 2-3 or 1-3 generations of neutrinos in the TBM basis. Other two cases are given by the additional rotation of the 1-3 or 1-2 generations of charged leptons with the TBM neutrinos. These four cases give different predictions among three mixing angles and the CP violating Dirac phase.

The rotation of the 2-3 generations of neutrinos in the TBM basis predicts $\sin^2 \theta_{12} < 1/3$ and the CP violating Dirac phase δ_{CP} to be $\pm(0.09\pi \sim 0.76\pi)$ for the NH case. It has a clear $\sin^2 \theta_{23}$ dependence, which will be testable in the future. If $\sin^2 \theta_{23}$ is smaller (larger) than $1/2$, $|\delta_{CP}|$ is larger (smaller) than $\pi/2$. For the IH case, the predicted bounds are reduced to $0.15\pi \lesssim |\delta_{CP}| \lesssim 0.73\pi$.

The rotation of the 1-3 generations of neutrinos in the TBM basis gives $\sin^2 \theta_{12} > 1/3$. The CP violating Dirac phase δ_{CP} is not constrained by the input of the present experimental data. It has also a remarkable $\sin^2 \theta_{23}$ dependence. If $\sin^2 \theta_{23}$ is smaller (larger) than $1/2$, $|\delta_{CP}|$ is smaller (larger) than $\pi/2$.

In the case of the rotation of the 1-3 and 2-3 charged lepton generations, $\sin^2 \theta_{23}$ is slightly larger than $1/2$ (Case III) or smaller than $1/2$ (Case IV). The CP violating Dirac phase is predicted in the narrow range of $\pm(0.35\pi \sim 0.60\pi)$ for both cases, which may be preferred in the recent T2K experiment. There is a clear $\sin^2 \theta_{12}$ dependence for δ_{CP} , which is contrast to Case I and Case II. In Case III, $|\delta_{CP}|$ is smaller (larger) than $\pi/2$ if $\sin^2 \theta_{12}$ is smaller (larger) than $1/3$. In Case IV, $|\delta_{CP}|$ is smaller (larger) than $\pi/2$ if $\sin^2 \theta_{12}$ is larger (smaller)

than $1/3$. Numerical results are same for the NH and IH cases.

Finally, in the special case of Case IV, θ_{13} is related with the Cabibbo angle such as $\sin \theta_{13} = \lambda/\sqrt{2} \simeq 0.16$. This relation can be derived from the framework of GUT, where the Yukawa matrices for the charged leptons and for the down-type quarks have the same origin. The CP violating phase is predicted to be in the region of $\pm(0.41\pi \sim 0.62\pi)$, which prefer the maximal CP violation.

Thus, the CP violating Dirac phase can distinguish the lepton flavor mixing patterns at T2K and NO ν A experiments in the future.

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